Token Machines for Multiport Interaction Combinators

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Game semantics [7, 1] and the Geometry of Interaction [6] are semantic frameworks centered around the dynamic, interactive behaviour of the interpreted object rather than on its static, extensional properties. The obtained semantics is often fully abstract, and its concreteness can be exploited in the definition of compilation schemes towards low-level languages, like in the Geometry of Synthesis [5].

Most game semantics models are defined as to reflect the behaviour of *sequential* languages—the classic notion of a strategy is after all deterministic and sequential. The interest around *concurrent* forms of game semantics is not at all new, however. Concurrency has been incepted into games both at the level of the *interpreting* object (e.g. [2, 13, 3]), and at the level of the *interpreted* object (e.g. [11]).

In the Geometry of Interaction, on the other hand, the way towards concurrency has been explored less extensively. If one looks at the most concrete incarnation of GoI, namely the so-called token-machines [4], there is however a very natural way to go parallel, namely considering machines whose internal state consists of possibly *many* tokens rather than just *one* [14, 9, 10]. If the interpreted language is sequential and deterministic, however, there is no way the various tokens could interact, and parallelism is then vacuous. If one injects a primitive for synchronization in the underlying calculus, on the other hand, multiple tokens become a necessity, and the model becomes adequate for quantum computation [9].

The authors are currently engaged in defining and studying a Geometry of Interaction model for *multiport interaction combinators* [12], a concurrent extensions of Lafont's interaction combinators [8]. The talk will be about this ongoing work.

Multiport Interaction Combinators Interaction combinators are a specific system of interaction nets in which cells are of three types, called γ, δ, and ε cells. The distinctive feature of interaction combinators is their universality: any other interaction net system can be encoded into interaction combinators (ICs) in a faithful way [8]. The geometry of interaction combinators is very simple and has nice properties, e.g., reduction turns a net into another having an equivalent GoI. As described in Mazza's thesis [12], the most satisfactory way of generalizing interaction nets to a framework capable of modeling concurrent computation consists of introducing *multiport* cells. Even in presence of multiports, noticeably, one can get universality by way of a system of combinators, the so-called *multiport interaction combinators* (MICs), which are actually mild variations of ICs in which δ cells can have more than one principal port.

Multitoken Machines for MICs Contrarily to ICs, giving a Geometry of Interaction model for MICs is quite challenging. The usual idea, which consists in seeing cells as stack transducers, does not seem to work. Indeed, the same multiport cell can behave very differently when interacting through either of its principal ports: this is after all the way nondeterminism is captured by MICs. It is thus necessary to instrument token machines so as to be able to handle this non-deterministic pairing mechanism. And in turn, this requires multiple tokens, since the discovery process underlying pairing cannot be done sequentially. We are currently working on soundness and adequacy proofs, on which we will report at the end of the talk.

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References

[1] Samson Abramsky, Radha Jagadeesan, and Pasquale Malacaria. Full abstraction for PCF. *Inf. Comput.*, 163(2):409–470, 2000.

[2] Samson Abramsky and Paul-André Melliès. Concurrent games and full completeness. In *Proceedings of LICS 1999*, pages 431–442, 1999.

[3] Simon Castellan, Pierre Clairambault, Silvain Rideau, and Glynn Winskel. Concurrent games. *CoRR*, abs/1604.04390, 2016.

[4] Vincent Danos and Laurent Regnier. Reversible, irreversible and optimal lambda-machines. *Theor. Comput. Sci.*, 227(1-2):79–97, 1999.

[5] Dan R. Ghica. Geometry of synthesis: a structured approach to VLSI design. In *Proceedings of POPL 2007*, pages 363–375, 2007.

[6] Jean-Yves Girard. Geometry of interaction 1: Interpretation of system F. *Logic Colloquium 88*, pages 221–260, 1989.

[7] J. M. E. Hyland and C.-H. Luke Ong. On full abstraction for PCF: I, II, and III. *Inf. Comput.*, 163(2):285–408, 2000.

[8] Yves Lafont. Interaction combinators. *Inf. Comput.*, 137(1):69–101, 1997.

[9] Ugo Dal Lago, Claudia Faggian, Ichiro Hasuo, and Akira Yoshimizu. The geometry of synchronization. In *Proceedings of CSL-LICS 2014*, pages 35:1–35:10, 2014.

[10] Ugo Dal Lago, Claudia Faggian, Benoît Valiron, and Akira Yoshimizu. Parallelism and synchronization in an infinitary context. In *Proceedings of LICS 2014*, pages 559–572, 2015.

[11] James Laird. Game semantics for higher-order concurrency. In *Proceedings of FSTTCS 2006*, volume 4337 of *LNCS*, pages 417–428, 2006.

[12] Damiano Mazza. *Interaction Nets: Semantics and Concurrent Extensions*. PhD thesis, Université de la Méditerranée and Università degli Studi Roma Tre, 2006.

[13] Paul-André Melliès and Samuel Mimram. Asynchronous games: Innocence without alternation. In *Proceedings of CONCUR 2007*, volume 4703 of *LNCS*, pages 395–411, 2007.

[14] Akira Yoshimizu, Ichiro Hasuo, Claudia Faggian, and Ugo Dal Lago. Measurements in proof nets as higher-order quantum circuits. In *Proceedings of ESOP 2014*, volume 8410 of *LNCS*, pages 371–391, 2014.